

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

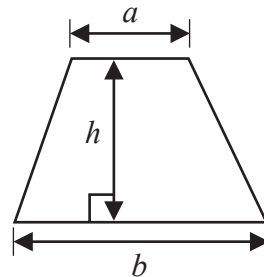
Sum to n terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

The quadratic equation

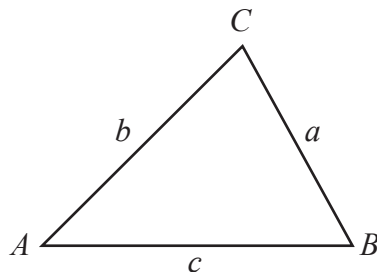
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium = $\frac{1}{2}(a+b)h$



Trigonometry



In any triangle ABC

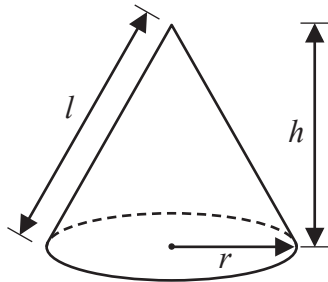
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

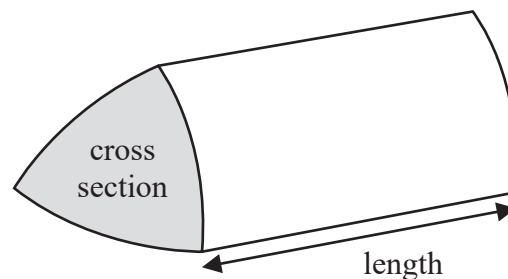
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



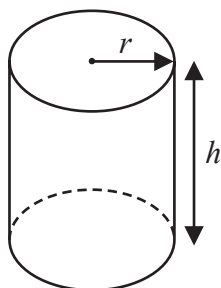
Volume of prism

= area of cross section $\times$ length



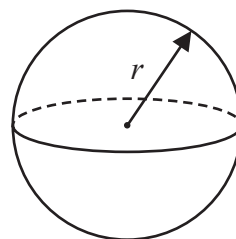
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



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Answer all TWENTY FOUR questions.

Write your answers in the spaces provided.

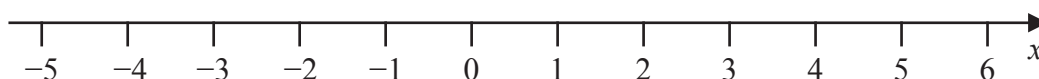
You must write down all the stages in your working.

- 1** Find the highest common factor (HCF) of 72 and 180
Show your working clearly.

.....
(Total for Question 1 is 2 marks)



- 2 (a) On the number line, show the inequality $-2 \leq x < 3$



(2)

- (b) Solve the inequality $5x - 9 < 21$

(2)

(Total for Question 2 is 4 marks)

- 3 Write 4500 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 3 is 3 marks)



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4 The table gives information about the length of time, in minutes, that each of 120 customers were in a cafe.

Length of time (t minutes)	Frequency
$0 < t \leq 20$	12
$20 < t \leq 40$	46
$40 < t \leq 60$	36
$60 < t \leq 80$	18
$80 < t \leq 100$	8

(a) Write down the modal class.

..... minutes
(1)

(b) Work out an estimate for the mean length of time these customers were in the cafe.

..... minutes
(4)

(Total for Question 4 is 5 marks)



5 The n th term of an arithmetic sequence is given by $8n - 5$

- (a) Work out the ratio of the 40th term of the sequence to the 60th term of the sequence.
Give your ratio in its simplest form.

.....
(3)

Here are the first five terms of a different arithmetic sequence.

3 8 13 18 23

- (b) Find an expression for the n th term of this sequence.

.....
(2)

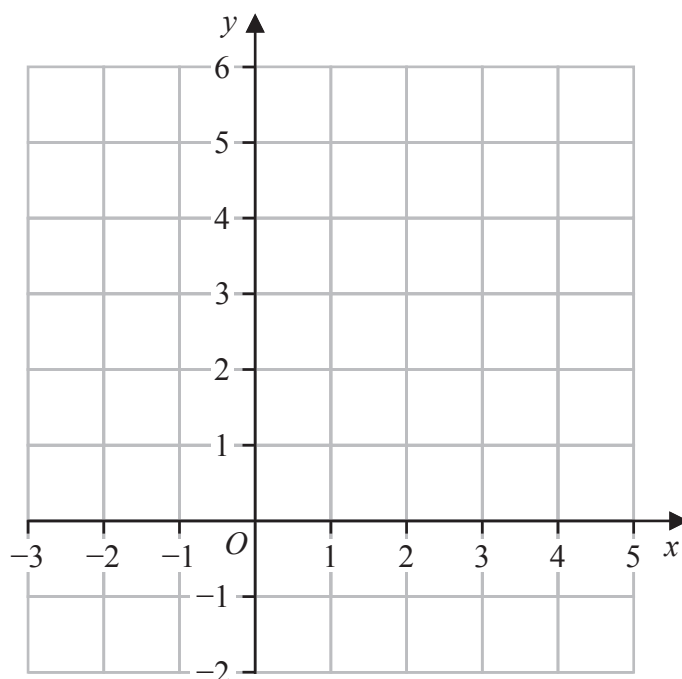
(Total for Question 5 is 5 marks)



6 (a) On the grid, draw the straight line with equation

(i) $x = -1$ (ii) $y = 4$ (iii) $y = x + 1$

Label each line with its equation.



(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$$x \geq -1 \quad y \leq 4 \quad y \geq x + 1$$

Label the region **R**

(1)

(Total for Question 6 is 4 marks)



- 7 Suha invests 25 000 yuan in a savings account for 3 years.
She gets 6% per year compound interest.

- (a) Work out how much money Suha will have in the savings account
at the end of 3 years.
Give your answer correct to the nearest yuan.

..... yuan
(3)

Yusuf invests a sum of money.
He gets 4.5% interest after 1 year.
The amount of interest Yusuf gets after 1 year is 180 lira.

- (b) Work out the amount of money that Yusuf invested.

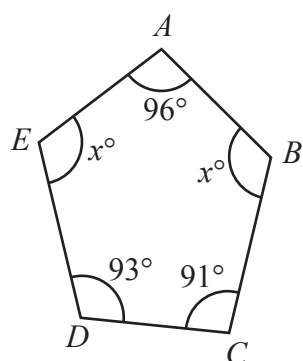
..... lira
(3)

(Total for Question 7 is 6 marks)



8 $ABCDE$ is a pentagon.

Diagram **NOT**
accurately drawn



angle $EAB = 96^\circ$ angle $BCD = 91^\circ$ angle $CDE = 93^\circ$ angle $DEA =$ angle $ABC = x^\circ$

Work out the value of x

$x =$

(Total for Question 8 is 3 marks)

- 9 (a) Write 0.000 56 in standard form.

.....
(1)

- (b) Work out $(8 \times 10^{52})^2$

Give your answer in standard form.

.....
(2)

- (c) Work out $\frac{2.8 \times 10^{-58}}{7 \times 10^{61}}$

Give your answer in standard form.

.....
(2)

(Total for Question 9 is 5 marks)



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10 The diagram shows a cylinder and a prism.

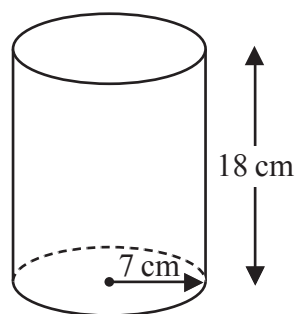
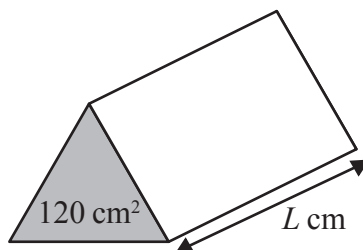


Diagram **NOT**
accurately drawn



The radius of the cylinder is 7 cm
The height of the cylinder is 18 cm

The area of the cross section of the prism, shown shaded in the diagram, is 120 cm^2
The length of the prism is $L \text{ cm}$

The volume of the cylinder is equal to the volume of the prism.

Work out the value of L
Give your answer correct to 3 significant figures.

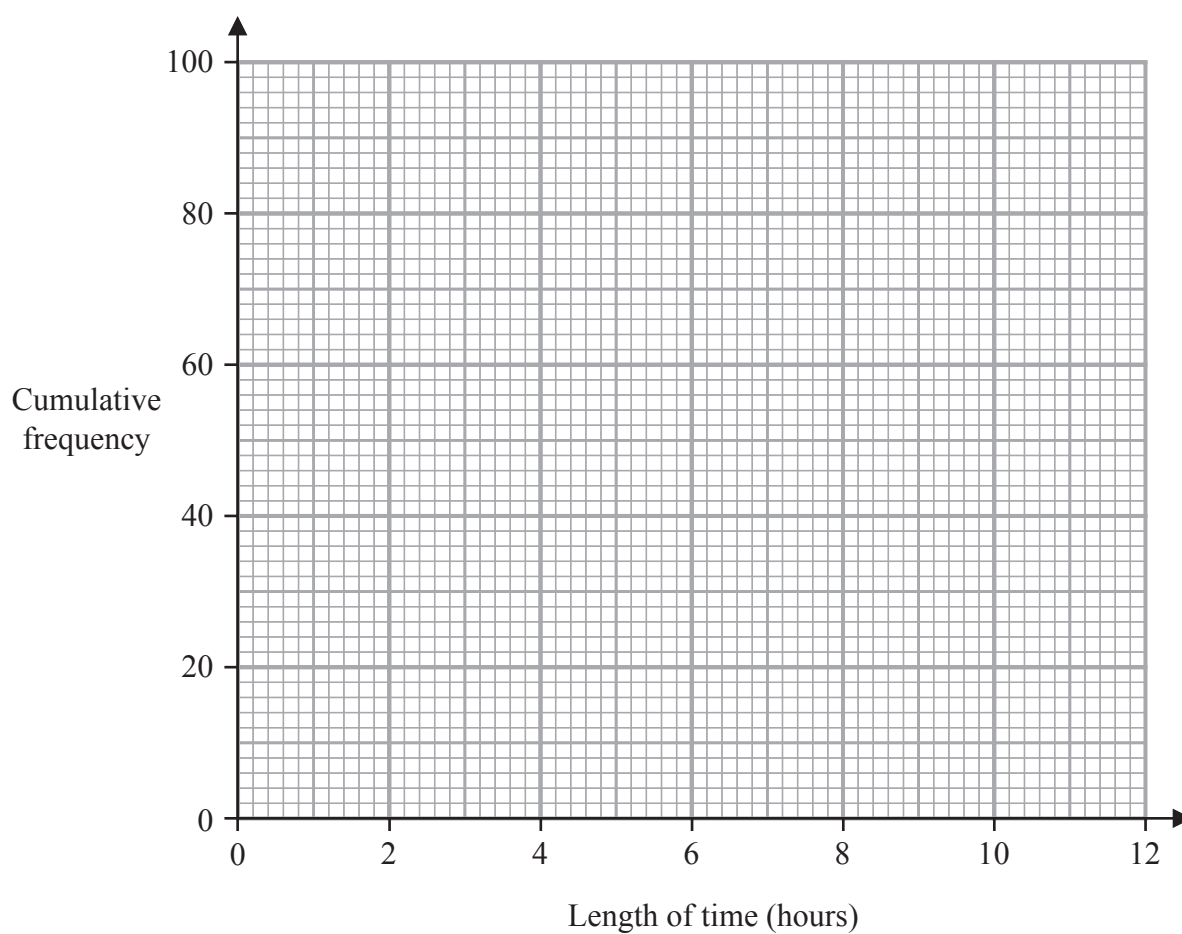
$L =$

(Total for Question 10 is 3 marks)

- 11 The cumulative frequency table shows information about the length of time that each of 100 people used their mobile phone on Friday.

Length of time (t hours)	Cumulative frequency
$0 < t \leq 2$	10
$0 < t \leq 4$	36
$0 < t \leq 6$	80
$0 < t \leq 8$	92
$0 < t \leq 10$	98
$0 < t \leq 12$	100

- (a) On the grid, draw a cumulative frequency graph for this information.



(2)



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- (b) Use your graph to find an estimate for the median length of time these people used their mobile phone.

..... hours
(1)

- (c) Use your graph to find an estimate for the number of these people who used their mobile phone for more than 7 hours.

.....
(2)

(Total for Question 11 is 5 marks)

- 12 Find the highest prime number that is a factor of 3.6×10^{24}

.....
(Total for Question 12 is 2 marks)



13 Solve the simultaneous equations

$$5x - 3y = 30$$

$$9x + 7y = 23$$

Show clear algebraic working.

$$x = \dots\dots\dots$$

$$y = \dots\dots\dots$$

(Total for Question 13 is 4 marks)



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14 Leo collects apples from three trees.
The incomplete table gives some information about the apples he collects from these trees.

Tree	Number of apples	Mean weight of apples (grams)	Total weight of apples (grams)
A	12	150	
B	20		2700
C		145	2610

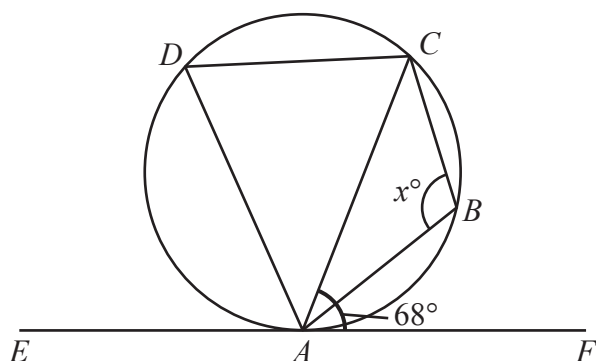
Work out the mean weight of all the apples Leo collects from these three trees.

..... grams

(Total for Question 14 is 4 marks)



Diagram **NOT**
accurately drawn



A, B, C and D are points on a circle.
 EAF is the tangent to the circle at A

$$\text{angle } CAF = 68^\circ \quad \text{angle } ABC = x^\circ$$

Work out the value of x
Give a reason for each stage of your working.

$$x = \dots\dots\dots$$

(Total for Question 15 is 4 marks)

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16 Make t the subject of $q = \sqrt{\frac{p-2t}{2+3t}}$

(Total for Question 16 is 4 marks)



- 17 A question was answered by 59 students.
The table shows their marks for this question.

Mark	Number of students
0	2
1	5
2	10
3	16
4	14
5	8
6	3
7	1

Work out the upper quartile for these marks.
Show your working clearly.

(Total for Question 17 is 2 marks)



18 The functions f and g are such that

$$f(x) = 4 + x \quad g(x) = 3x^2 + 2$$

(a) Find $g(4)$

.....
(1)

(b) Find $f^{-1}(7)$

.....
(2)

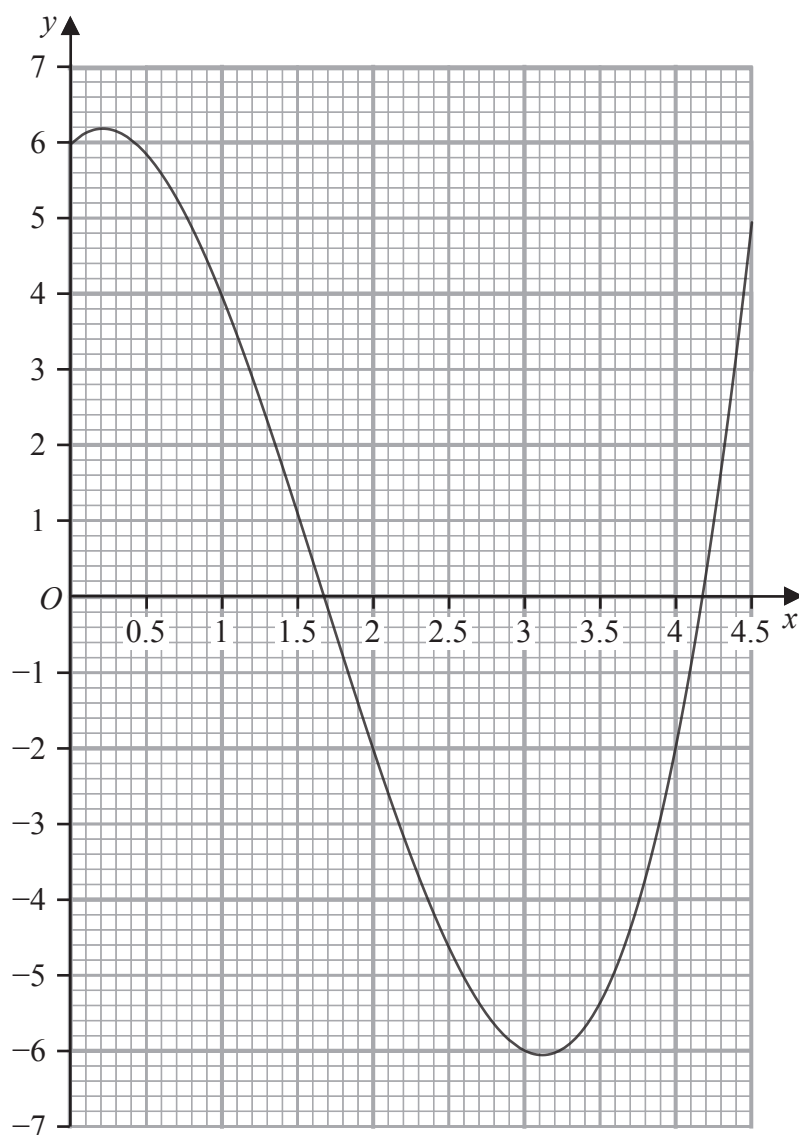
(c) Express $gf(x)$ in the form $ax^2 + bx + c$ where a , b and c are integers.

$gf(x) =$
(2)

(Total for Question 18 is 5 marks)



19 The curve with equation $y = f(x)$ is drawn on a grid.



- (a) By drawing a suitable straight line (at least 3 cm long) on the grid, work out an estimate for the gradient of the curve at the point where $x = 1$. Show your working clearly.

.....
(2)



- (b) By drawing a suitable straight line on the grid, find the solutions of the equation $x = 3 + \frac{1}{2}f(x)$
- Show your working clearly.

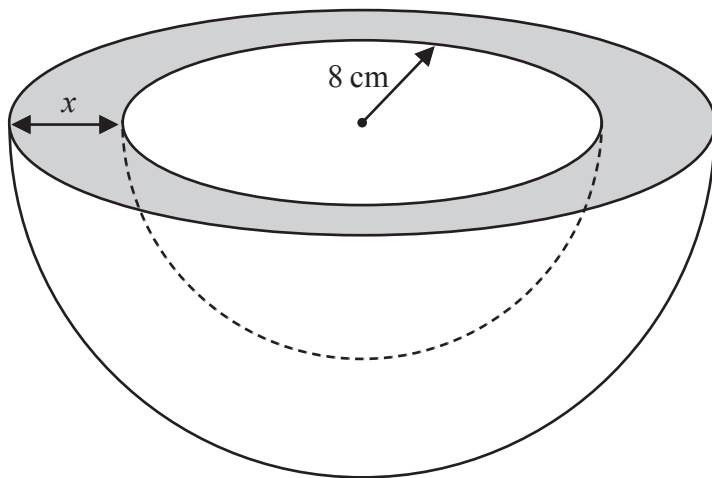
.....
(3)

(Total for Question 19 is 5 marks)



- 20 The diagram shows a solid of uniform thickness, x cm
The solid is formed by removing a small hemisphere from a larger hemisphere.

Diagram **NOT**
accurately drawn



The radius of the smaller hemisphere is 8 cm
The **total** surface area of the solid is $394.75\pi \text{ cm}^2$

Work out the volume of the solid.
Give your answer correct to 3 significant figures.

Show your working clearly.

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..... cm³

(Total for Question 20 is 6 marks)



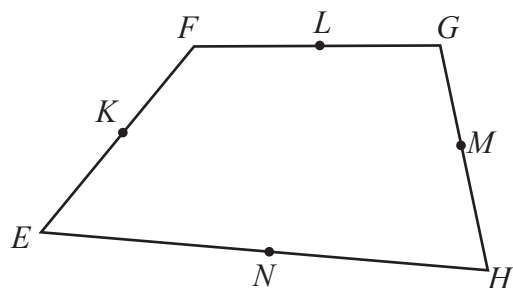


Diagram **NOT**
accurately drawn

$EFGH$ is a quadrilateral.

K , L , M and N are midpoints of the sides of the quadrilateral.

$$\vec{EF} = 2\mathbf{a} \quad \vec{FG} = 2\mathbf{b} \quad \vec{GH} = 2\mathbf{c}$$

Use a vector method to prove that $KLMN$ is a parallelogram.

(Total for Question 21 is 4 marks)



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22 Work out the sum of all the integers from 1 to 1000 that are not multiples of 3

(Total for Question 22 is 4 marks)



- 23 The diagram shows a shape made from a semicircle and three sides of a rectangle.

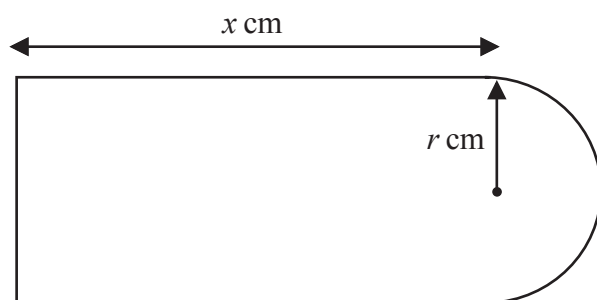


Diagram **NOT**
accurately drawn

The semicircle has radius r cm

The rectangle has length x cm and width $2r$ cm

The perimeter of the shape is 200 cm

Work out the value of r for which the area of the shape is a maximum.
Give your answer correct to the nearest whole number.

Show your working clearly.

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$r =$

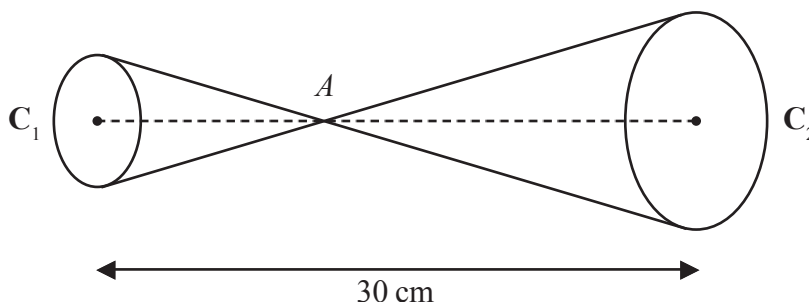
(Total for Question 23 is 6 marks)

Turn over for Question 24



24 The diagram shows two cones C_1 and C_2

Diagram **NOT**
accurately drawn



The vertex of C_1 coincides with the vertex of C_2 at the point A
 C_1 and C_2 have the same axis of symmetry.

$$\text{height of } C_1 + \text{height of } C_2 = 30 \text{ cm}$$

$$\text{total surface area of } C_1 : \text{total surface area of } C_2 = 4 : 9$$

$$\text{volume of } C_1 = 23.04\pi \text{ cm}^3$$

Work out the radius of the base of C_2
 Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

TOTAL FOR UNIT IS 100 MARKS

